Lösungen AB01

1a) $f(x) = x^2 \cdot \sin(x)$

$$f'(x) = \underline{\underline{2x \cdot \sin(x) + x^2 \cdot \cos(x)}}$$

b) $f(x) = x \cdot (x-1)^{-1}$ Alternativ: Quotientenregel!

$$f'(x) = 1 \cdot (x-1)^{-1} + x \cdot \left[(x-1)^{-1}\right]'$$

Kettenregel!

$$= 1 \cdot (x-1)^{-1} + x \cdot (-1) \cdot (x-1)^{-2} \cdot 1$$

$$= \frac{1}{x-1} + \frac{-x}{(x-1)^2}$$

$$= \frac{x-1}{(x-1)^2} - \frac{x}{(x-1)^2}$$

$$= \frac{x-1-x}{(x-1)^2} = \underline{\underline{-\frac{1}{(x-1)^2}}}$$

c) $f(x) = (3x^2+x-2) \cdot \sqrt{x}$ $\leftarrow = x^{\frac{1}{2}}$

$$f'(x) = (6x+1)\sqrt{x} + (3x^2+x-2) \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$= \underline{\underline{(6x+1)\sqrt{x} + (3x^2+x-2) \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}}}}$$

2.) a) $f(x) = \dfrac{x^2+1}{2x+3}$

$$f'(x) = \frac{(2x+3)\,2x - (x^2+1) \cdot 2}{(2x+3)^2}$$

$$= \frac{4x^2+6x-2x^2-2}{(2x+3)^2}$$

$$= \frac{2x^2+6x-2}{(2x+3)^2}$$

b) $$f(x) = \frac{x^2+2x-1}{x-1}$$

$$f'(x) = \frac{(x-1)\cdot(2x+2) - (x^2+2x-1)\cdot 1}{(x-1)^2}$$

$$= \frac{2x^2+2x-2x-2-x^2-2x+1}{(x-1)^2}$$

$$= \frac{x^2-2x-1}{(x-1)^2}$$

c) $$f(x) = \frac{2x-1}{(x-1)^2} = \frac{2x-1}{x^2-2x+1}$$

$$f'(x) = \frac{(x^2-2x+1)\cdot 2 - (2x-1)(2x-2)}{(x^2-2x+1)^2}$$

$$= \frac{2x^2-4x+2-(4x^2-4x-2x+2)}{(x^2-2x+1)^2}$$

$$= \frac{2x^2-4x+2-4x^2+6x-2}{(x-1)^4}$$

$$= \frac{-2x^2+2x}{(x-1)^4} = \frac{-2x(x-1)}{(x-1)^4}$$

$$= \frac{-2x}{(x-1)^3}$$

3a) $f(x) = (2+x)^5$; $a(x) = x^5$; $i(x) = 2+x$

b) $f(x) = \sin(1-\sqrt{x})$; $a(x) = \sin(x)$

$i(x) = 1-\sqrt{x}$

c) $f(x) = 2^{2x+1}$; $a(x) = 2^x$

$i(x) = 2x+1$

4a) $f(x) = \sqrt{2x^2+x-3}$

$a(x) = \sqrt{x} = x^{1/2}$; $i(x) = 2x^2+x-3$

$a'(x) = \frac{1}{2}x^{-1/2}$; $i'(x) = 4x+1$

$$f'(x) = \frac{1}{2}\left(2x^2+x-3\right)^{-1/2} \cdot (4x+1)$$

$$= \underline{\underline{\frac{4x+1}{2\sqrt{2x^2+x-3}}}}$$

b) $f(x) = \sin(x^2-3x)$

$a(x) = \sin(x)$; $i(x) = x^2-3x$

$a'(x) = \cos(x)$; $i'(x) = 2x-3$

$$f'(x) = \cos(x^2-3x) \cdot (2x-3)$$

Beim Weiterrechnen besser vor den Cosinus schreiben, um klar zu sehen, dass der Faktor nicht innerhalb der Cosinusfunktion steht!

c) $f(x) = (2x - \sin(x))^3$

$a(x) = x^3$; $i(x) = 2x - \sin(x)$

$a'(x) = 3x^2$; $i'(x) = 2 - \cos(x)$

$$f'(x) = \underline{\underline{3\,(2x - \sin(x))^2 \cdot (2 - \cos(x))}}$$

5a) $f(x) = \sqrt{\frac{3-2x}{2x-3}} = \sqrt{\frac{-(2x-3)}{2x-3}} = \sqrt{-1}$ ↯

Sorry, war so nicht gedacht...

b) $f(x) = \sin(2x+3) \cdot x^2$

$f'(x) = [\sin(2x+3)]' \cdot x^2 + \sin(2x+3) \cdot 2x$

$= \underline{\underline{2 \cdot \cos(2x+3) \cdot x^2 + 2x \cdot \sin(2x+3)}}$

NR: $h(x) = \sin(2x+3)$

$a(x) = \sin(x)$; $i(x) = 2x+3$

$a'(x) = \cos(x)$; $i'(x) = 2$

$h'(x) = \cos(2x+3) \cdot 2$

c) $f(x) = \frac{\sin(3x-5)}{(4x-3)^5}$

$$f'(x) = \frac{(4x-3)^5 \cdot [\sin(3x-5)]' - \sin(3x-5) \cdot [(4x-3)^5]'}{[(4x-3)^5]^2}$$

$$= \frac{(4x-3)^5\, 3 \cdot \cos(3x-5) - \sin(3x-5) \cdot 20\,(4x-3)^4}{(4x-3)^{10}}$$

$$= \underline{\underline{\frac{(4x-\cancel{3})^4\,[3 \cdot (4x-3) \cdot \cos(3x-5) - 20 \cdot \sin(3x-5)]}{(4x-3)^{\cancel{10}\,6}}}}$$

NR1: $h_1(x) = (4x-3)^5$

$a(x) = x^5$; $i(x) = 4x-3$

$a'(x) = 5x^4$; $i'(x) = 4$

$h_1'(x) = 5(4x-3)^4 \cdot 4$

$= 20(4x-3)^4$

NR2: $h_2(x) = \sin(3x-5)$

$a(x) = \sin(x)$; $i(x) = 3x-5$

$a'(x) = \cos(x)$; $i'(x) = 3$

$h_2'(x) = \cos(3x \cdot 5) \cdot 3$